

## Cluster Analysis of the Nonlinear Evolution of Large-Scale Structure in an Axion/Gravitino/Photino-Dominated Universe

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(Received 23 May 1983)

The authors consider nonlinear evolution of the inflationary scale-free spectrum of adiabatic density perturbations in (1) a neutrino-dominated universe (sharp short-wavelength cutoff), and (2) an axion-, gravitino-, or photino-dominated universe (some small-scale power). In (2) galaxy formation begins long before the present (as defined by covariance function), resolving a possible problem in (1). Both models are found to be acceptable upon cluster analysis. The authors believe that a new picture, based on (2), merits consideration.

PACS numbers: 98.50.Eb, 11.30.Pb, 14.80.Pb

The dynamically indicated but unseen "missing mass" in astrophysical systems has been an unsolved problem for half a century.<sup>1,2</sup> Nucleosynthesis arguments<sup>3</sup> make nonbaryonic particles an attractive hypothesis, with massive neutrinos a leading candidate.

Damping in neutrino-dominated universes<sup>4-6</sup> removes density perturbations smaller than  $\lambda_c = 2\pi/k_c$ ; for  $m_\nu \sim 30$  eV such a scale is characteristic of superclusters. Thus the primordial power spectrum is preserved up to a sharp cutoff. Gravitational collapse in such a universe is anisotropic and may lead to formation of galaxies from fragmentation of these structures.<sup>7</sup> We call this the adiabatic (A)<sup>8</sup> theory. Simulations of the A theory with massive neutrinos indicate that galaxy/halo formation is possible and the structure compares reasonably well with observed large-scale structure of the universe.<sup>9-20</sup> A problem has arisen in the analysis of structure in the A theory, however. The value of  $\gamma$  in the two-point correlation function  $\xi(R) \propto R^{-\gamma}$  attains its observed value  $\sim 1.8$  only for a short time after the collapse of structure, but galaxies must have formed at an early epoch.<sup>15-17</sup> Therefore one cannot simultaneously have the proper slope  $\gamma$  and have galaxies form long before the present, if this result is accepted. However, it should be mentioned that this result was obtained by ignoring dissipation, which could affect it. The ob-

served  $\gamma$  is determined from galaxy counts, but the simulations to date include only "neutrinos."

Preceding this, there were numerical simulations<sup>21,22</sup> of the hierarchical clustering theory (HC), in which galaxies form by coalescence of smaller subunits, and in turn cluster to form larger ones.<sup>23</sup> In this case it is assumed that the primordial power spectrum of density perturbations was undamped and so retains its primordial slope. HC simulations have been able to reproduce the  $\gamma \sim 1.8$  value<sup>21</sup> but only at high amplitudes.<sup>22</sup> It seems that the proper large-scale structure is not produced,<sup>20,24</sup> in that statistical measures show too little "filamentary" character, as we will explain later.

Gravitinos<sup>25-27</sup> and photinos<sup>28</sup> are also attractive candidates for missing mass. Decoupling earlier, they have a lower present number density than relic neutrinos and may have larger masses and cluster in smaller systems. This is an attractive property if observations show that hidden mass exists in dwarf galaxies<sup>29,30</sup> or in galactic disks.<sup>31</sup> Axions are bosons "created cold" which in spite of low mass may dominate galaxies and the universe through their high number density.<sup>32,33</sup>

It is usually assumed that primordial density perturbations followed a power law

$$|\delta_k|^2 \propto k^n, \quad (1)$$

where  $\delta_k$  are the Fourier components of density.

One can then write the density contrast on scale  $k_i$  as

$$(\delta_\rho/\rho)_{k_i}^2 \propto \int_0^{k_i} k^2 |\delta_k|^2 dk, \quad (2)$$

or

$$(\delta_\rho/\rho)_{k_i} \propto k^{3/2+n/2}. \quad (3)$$

Undamped perturbations in a neutrino universe have almost uninterrupted growth, so that (in the linear regime) the slope of the power spectrum is preserved for  $0 < k < k_c$ . In an axion-, gravitino-, or photino-dominated universe (AGP) there exists a time for which they are nonrelativistic but do not yet dominate. Perturbations entering the horizon during this period grow very little until they dominate. The resulting power spectrum is like the neutrino case for  $0 < k < k_b$ , but is "bent" to about  $n-4$  for  $k_b < k < k_c$ . The scale  $k_b$  is determined by the horizon at the time when nonrelativistic species come to dominate, and for reasonable values of the present mass density and microwave temperature corresponds to supercluster scales, as does  $k_c$  for neutrinos. The free-stream damping scale  $k_c$  may be of galactic scales for gravitinos or photinos but is much smaller for axions or other cold particles. The shape of the power spectrum (except the value of  $k_c$ ) is very general, and will arise for any sort of relic particle which is nonrelativistic before it begins to dominate the mass density of the universe. (The generic neutrino shape arises for "hot" particles, for which  $k_b \sim k_c$ .) For numerical results of calculations of these spectra, see the work of Bond, Szalay, and Turner.<sup>26</sup>

Thus on large scales AGP universes possess a similar structure which resembles neither that of the A nor the HC theory (unless  $n \geq 3$ , in which case it resembles HC).<sup>33</sup> If  $n < 1$  perturbations diverge on large scales without an *ad hoc* cutoff. The  $n=1$  spectrum<sup>34-36</sup> is scale-free and emerges naturally from consideration of some inflationary universe schemes,<sup>37-41</sup> and we adopt this spectrum for our simulation.

It is sometimes stated, on the basis of (3) that for  $n=1$  (bent to  $-3$ ) all scales  $k_b < k < k_c$  go nonlinear at once. However, (2) increases logarithmically. This increase is accentuated by the fact that the freezeout is not perfect, and power on galactic scales may grow by a small amount.<sup>26,33</sup> This means that galactic scales collapse at a cosmic expansion factor six or more times smaller than that of the collapse of supercluster scales for our assumed spectrum.

The effective spectral index on small scales is

then  $n_e \gtrsim n-4$ . This is in accord with the conclusion that  $n_e < -1$  which is based on cooling of gas to form galaxies.<sup>42</sup> For  $n_e \sim -3$ , energy per unit mass in bound systems is proportional to the size of the systems<sup>43,44</sup> which agrees with observation.<sup>45</sup>

We have simulated the nonlinear evolution of structure in an AGP universe using the "bent" power spectrum described earlier as the initial condition for a cloud-in-cell (CIC)<sup>46</sup> gravitational clustering code. This code is an outgrowth of earlier work.<sup>10-13</sup> It uses one cloud per cell on a  $32^3$  mesh, but higher-resolution work is planned. In any case it will not be possible to resolve both  $k_b$  and  $k_c$  in such a code; we optimize the strength of the method, and study AGP universes in large-scale structure. We compare to an A model using the same code, and to the observations. The power spectrum of the two models was initially the same up to a value ( $k_b$  in AGP =  $k_c$  in A) beyond which there was a sharp cutoff in A and a bend to  $n=-3$  in AGP.

We wish to emphasize that our conclusions depend on two approximations: (1) The gradual bend from  $n=1$  to  $n=-3$  is approximated by a sudden bend. (2) CIC codes accurately model large-scale collective modes but do not follow small-scale dynamics. It should be mentioned in regard to (2) that direct  $N$ -body methods follow small scales at the cost of introducing spurious noise on these time scales. This CIC code follows the dynamics accurately to  $k \sim 2k_b$ , and with decreasing accuracy to  $k \sim 4k_b$ . Thus we confine our attention to large-scale structures only. Also, smaller-scale structure will be affected by gas-dynamical processes.

In AGP the covariance function<sup>47</sup>  $\xi(R)$  steepens as in the A model, but the AGP model attains the observed value  $\sim 1.8$  just about at the time of structure formation on scale  $k_b$ . The amplitude of  $\xi$  at this time suggests that  $\lambda_b \sim 40$  Mpc (pc  $\equiv$  parsec). If the covariance function is correctly calculated in the absence of hydrodynamics, this says that large-scale structure is now pancaking, but galaxy formation may have proceeded at  $Z \geq 5$  which removes a difficulty associated with the A model. Structures collapsing at this epoch could have radii  $\sim 100$  kpc and densities  $\sim 10^{-25}$  g cm<sup>-3</sup>, interestingly close to the characteristics<sup>2</sup> of galactic halos.

The density of an  $x$ -dominated universe, where  $x$  denotes a hypothetical particle, may be written as  $\Omega h^2 = 7 \times 10^{-2} (m_x/1 \text{ eV}) g^{-1} g_x$ , where  $\Omega$  is the ratio of density to critical density,  $h$  is the Hub-

ble constant in units of  $100 \text{ km s}^{-1} \text{ Mpc}^{-1}$ ,  $m_x$  is our particle mass,  $g_x$  is the total effective number of degrees of freedom in all relativistic species at  $x$  decoupling, and  $g_x$  is the effective number of degrees of freedom in  $x$ .<sup>25,26</sup> The scale  $\lambda_b$  is  $(510 \text{ Mpc})(m_x/1 \text{ eV})^{-1} g_x g^{-1}$ .<sup>26</sup> We therefore have the relation  $\lambda_b \sim (36 \text{ Mpc})/\Omega h^2$ ; our  $\sim 40 \text{ Mpc}$  result from the simulation is compatible within the uncertainties to the observational bound  $\Omega h^2 < 2$ . This resolves a possible scaling problem<sup>16,48</sup> which existed in neutrino (A) simulations, in that it seemed that  $\Omega h^2$  would exceed this bound if galaxy formation proceeded at  $Z \geq 4$ , as observations seem to require. The value  $40 \text{ Mpc}$  is characteristic of observed superclusters.

Visual inspection of particle position plots shows that the AGP model has a coherent structure on the scale  $k_b$ , in common with A models. Filamentary structures and voids are common in the AGP model, but there are some condensations in low-density regions. The AGP structure is more fragmented in appearance than the A structure. Galaxies may form in the voids but could possibly survive as dwarf galaxies and escape observation. Most matter collects in the coherent structure.

The function  $\xi$  cannot distinguish a nonlinear filamentary structure from an unclustered population,<sup>49</sup> but the method of cluster analysis is able to do so.<sup>50</sup> In our approach two points are considered "neighbors" if their separation is less than neighborhood radius  $r_n$ . The principle "any neighbor of my neighbor is a neighbor of mine" is used to define connected structures.

The mass-weighted differential multiplicity function  $f(n)$  is defined here as the fraction of galaxies in systems of membership  $n - dn$  to  $n$ . Previous studies showed that a simple hierarchical clustering model<sup>51</sup> strongly disagreed with observation in this respect. "A" simulations agreed rather well. It is not yet certain whether HC numerical simulations agree.

A characteristic size may be defined as the maximum distance between any two members of the same structure for given  $r_n$ . When a single structure spans the system, we say that percolation has taken place at neighborhood radius  $r_c$ .<sup>52</sup> We scale the radius to the radius of a sphere containing on average one particle.

There must be some unclustered primordial population. We exclude from consideration all particles not connected at neighborhood radius  $0.89$ , the virial radius for two initial diagonal neighbors in a CIC code; this conservatively re-

jects (15–25)% of material as pregalactic.

The percolation parameter  $B_c = (4\pi/3)r^3$  is found to be 1.15 in observed samples, with an error of a factor of 2 possible as a result of magnitude-limited samples and local density enhancement.<sup>50</sup> It is easily possible to fit this value and have  $\gamma \sim 1.8$  in both A and AGP models. The simple hierarchical clustering model has  $B_c \sim 5-11$ , and a Poisson distribution has  $B_c \sim 2.7$ .<sup>50</sup> For the A model we find a range 0.44 to 2.01, and for the AGP model 1.01 to 2.01.

We have studied the mass-weighted differential multiplicity function as it varies with time and  $r_n$  in both models. We find that the distribution of small, intermediate, and large systems is acceptable at the same time that the covariance function and percolation parameter are also.

It should be mentioned that while large-scale perturbations (above dipole) of the microwave background are of amplitude  $\sim 2 \times 10^{-5}$  in a neutrino-dominated A model,<sup>53,54</sup> amplitude  $\sim 3 \times 10^{-6}$  is expected in this AGP model,<sup>27</sup> which is far below current sensitivity, and thus consistent with upper limits at present.

To sum up, it seems that the AGP universe shares many properties of the A universe which agree with observation. In addition, a possible problem of the A (neutrino) model is solved. The AGP universe with the scale-free primordial density spectrum has intensive galaxy formation at  $Z \geq 5$  (depending on details of the particle physics), comfortably early enough to account for galaxy/quasar evolution, as compared with the time that the covariance function attains slope  $\gamma \sim 1.8$ , and is compatible with constraints on  $\Omega h^2$ .

We stress that our conclusions depend on the power spectrum used, not the specific particles. Nevertheless, we see here strong support for the structure formation process in an axion-, gravitino-, or photino-dominated universe. Galaxy formation proceeds from collapse of small-scale perturbations, as in the HC theory, but large-scale coherent structure forms as in A. The details of such a universe merit further study.

We have benefited from conversations with J. Barrow, J. R. Bond, M. Davis, C. Frenk, J. Ipser, J. Peebles, M. Rees, D. Sciama, P. Sikivie, M. Turner, S. White, and Ya. Zel'dovich. A. Szalay and S. Hawking deserve special thanks for suggesting that AGP simulation might prove interesting. One of us (A.L.M.) was the recipient of an International Research and Exchanges Board Fellowship for a research visit

to the Soviet Union. This work was supported in part by the National Science Foundation through Grant No. AST-8109485. We are grateful for large amounts of computer time provided at the University of Pittsburgh and at the Institute of Cybernetics, Tallinn, Estonia, U.S.S.R.

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